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A Computer Program to Calculate Fission Product Release from a Reactor Containment Building for Arbitrary Radioactive Decay Chains

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LEAF: A COMPUTER PROGRAM TO CALCULATE FISSION PRODUCT RELEASE FROM A REACTOR CONTAINMENT BUILDING FOR ARBITRARY RADIOACTIVE DECAY CHAINS

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ABSTRACT

This report describes an analytic containment building model that is used for calculating the leakage into the environment of each isotope of an arbitrary radioactive decay chain. The model accounts for the source, the buildup, the decay, the cleanup, and the leakage of isotopes that are gas-borne inside the containment building.

I. INTRODUCTION

The sources of the radioactive materials that are needed for the calculation of the consequences of postulated reactor accidents are obtained from estimates of the leakage of fission products from the reactor containment building. These estimates are obtained from a mathematical model of the reactor containment building that accounts for the source, the decay, the cleanup, and the leakage of each radionuclide in the building. A containment building model, which assumes that the gas in the building consists of a single, well mixed volume, is described in the Reactor Safety Study.[1] We have used a similar model[2,3] to estimate the time-dependent release of ^{131}I from the containment building of a High-Temperature Gas-Cooled Reactor (HTGR) during the Loss of Forced Circulation (LOFC) accident. The containment building model of References 2 and 3 is useful only for single isotopes because the radioactive decay chains are not included.

In this report we describe an analytic containment building model that is used for calculating the leakage into the environment of each isotope of an arbitrary radioactive decay chain. The model accounts for the source, the buildup, the decay, the cleanup, and the leakage of the isotopes that are gas-borne inside the containment building. In the model, the source of an isotope inside the containment building (which is the result of leakage from the reactor vessel), its removal rate by the containment cleanup system, and its leakage from the containment building are all assumed to be constant during short time intervals. We assume, as is done in Ref. 1, that the gas inside the containment building is well mixed and all in one compartment. Natural deposition of gas-borne isotopes onto surfaces internal to the containment building was not included in the model.

Even though we use this containment building model to estimate the time-dependent release of fission products to the environment for postulated HTGR accidents, the model is quite general and can be used for other types of reactors.

II. LEAF MODEL EQUATIONS

We consider a system shown in Fig. 1, composed of a reactor vessel emitting a source of radioactive materials $\underline{S}$, surrounded by a containment building which leaks at a rate L (s^{-1}). Inside the containment building there is a cleanup system filter having a cleanup rate V (s^{-1}). Let $\underline{N}$ be the amount of an isotope of a chain in the containment building, and $\underline{F}$ the total amount of the isotope absorbed on the filter. $\underline{N}$, $\underline{F}$, and $\underline{S}$ are vectors, the elements of which are the values of the individual species in the chain.

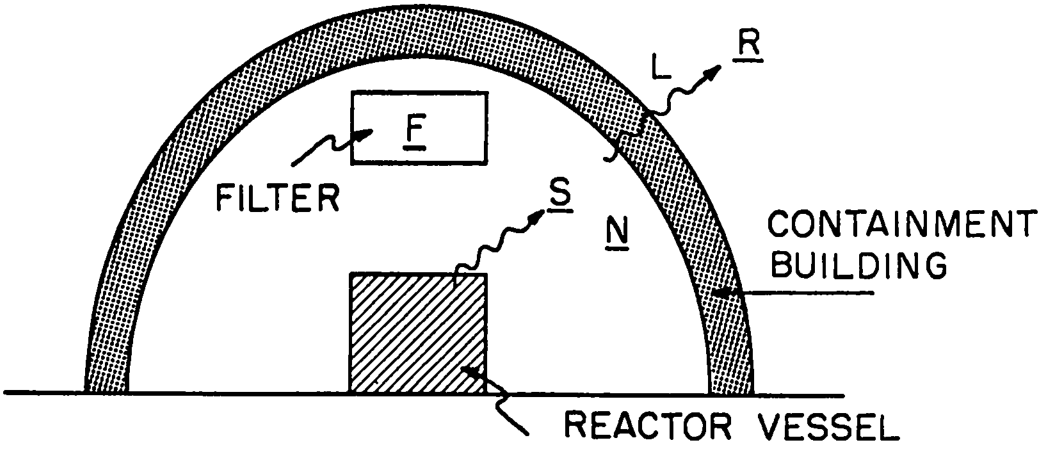

Fig. 1. LEAF containment building model.

Denote $\bar{\lambda}$ the decay chain matrix, and L and V the diagonal leak and filter cleanup rate matrices. The negative off-diagonal elements of $\bar{\lambda}$ include the branching ratio factors; the diagonal elements of $\bar{\lambda}$ are positive. Noble gases will not be filtered by the cleanup system. This fact is represented by a matrix $\bar{\delta}$ of the form

$$\bar{\delta}_{ij} = \mu_i \, (1-\delta_{ij}), \tag{1}$$

where

$$\mu_i = \begin{cases} 1 \text{ if the } i^{th} \text{ isotope is a noble gas} \\ 0 \text{ otherwise} \end{cases} \tag{2}$$

and δ_{ij} is the Kronecker delta, defined by

$$\delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j. \end{cases} \tag{3}$$

Defining the matrices λ, λ^*, and Λ by

$$\begin{aligned} \lambda &= \bar{\lambda} - \lambda^*, \\ \lambda^* &= \bar{\delta} \otimes \bar{\lambda} \, , \\ \Lambda &= \bar{\lambda} + V + L, \end{aligned} \tag{4}$$

where $\otimes$ denotes the Cartesian product, then the LEAF model equations are written as

$$\begin{aligned} \frac{d\underline{N}}{dt} &= -\Lambda\underline{N} - \lambda^* \, \underline{F} + \underline{S}, \\ \frac{d\underline{F}}{dt} &= V\underline{N} - \lambda\underline{F}. \end{aligned} \tag{5}$$

If we define new vectors $\underline{X}$ and $\underline{s}$ by

$$\underline{X} = \begin{pmatrix} \underline{N} \\ \underline{F} \end{pmatrix}, \quad \underline{s} = \begin{pmatrix} \underline{S} \\ 0 \end{pmatrix}, \tag{6}$$

and a supermatrix A by

$$A = \begin{pmatrix} -\Lambda & -\lambda^* \\ V & -\lambda \end{pmatrix}, \tag{7}$$

then Eq. (5) may be rewritten as

$$\frac{d\underline{X}}{dt} = A \, \underline{X} + \underline{s}. \tag{8}$$

We assume that the matrix A and the vector $\underline{s}$ are constant over the time interval $(0,\tau)$.

We are interested in calculating the integrated release to the environment by leakage from the containment building. This is given by

$$\underline{R}\,(\tau) = \int_0^\tau dt' L(t')\underline{N}(t') \qquad (9)$$

$$= \bar{L} \int_0^\tau dt' \underline{N}(t'),$$

where $\bar{L}$ is the average leakage in the time interval τ, defined by

$$\bar{L} = \frac{1}{2}\Big(L(0) + L(\tau)\Big)\ . \qquad (10)$$

If we define

$$\underline{Y}(\tau) = \begin{pmatrix} \underline{R}(\tau) \\ 0 \end{pmatrix}\ ,\ B = \begin{pmatrix} \bar{L} & 0 \\ L & 0 \end{pmatrix}, \qquad (11)$$

then Eq. (9) becomes

$$\underline{Y}(\tau) = B \int_0^\tau ds\ \underline{X}\ (s). \qquad (12)$$

As we shall prove in detail below, if the matrix A is constant in the time interval $(0,\tau)$, the solutions to Eqs. (8) and (12) are given by

$$\underline{X}\ (\tau) = \underline{X}(0) + \tau D(A\tau)\ \Big[A\ \underline{X}(0) + \underline{s}\Big]\ , \qquad (13)$$

and

$$\underline{Y}\ (\tau) = B\ [\tau D(A\tau)\ \underline{X}(0) + \tau^2\ Z(A\tau)\ \underline{s}]\ , \qquad (14)$$

where the matrix operators $D(C)$ and $Z(C)$ for $C = A\tau$ are evaluable using the methods of Ref. 4.

III. ANALYTIC SOLUTIONS TO THE LEAF MODEL EQUATIONS

Because the matrix A is constant in the time interval $(0,\tau)$, then following the Volterra method of the multiplicative integral,[4,5] we may construct the matricant

$$\Omega_0^\tau\ (A) = \exp\left[\int_0^\tau A(s)ds\right] = \exp\ (A\tau). \qquad (15)$$

The solution to Eq. (8) is given by

$$\underline{X}(\tau) = \Omega_0^\tau\ (A)\ \underline{X}(0) + \int_0^\tau dt'\ K\ (\tau,t')\underline{s}(t'), \qquad (16)$$

where

$$K(\tau,t') \equiv \Omega_0^\tau\ (A)\ \left[\Omega_0^{t'}\ (A)\ \right]^{-1}. \qquad (17)$$

As is readily proved,[4] both the matrix A and $e^{A\tau}$ are non-negative.

Substituting Eq. (15) into Eqs. (16) and (17) gives

$$\underline{X}\ (\tau) = e^{A\tau}\ \underline{X}(0) + e^{A\tau} \int_0^\tau dt' e^{-At'}\ \underline{s}(t'). \qquad (18)$$

Assuming that $\underline{s}(t') = \underline{s}$ is constant over the interval $(0,\tau)$, Eq. (18) becomes

$$\underline{X}(\tau) = e^{A\tau}\, \underline{X}(0) + A^{-1}(e^{A\tau} - I)\underline{s}. \tag{19}$$

Defining the matrix operator D(C) by[4]

$$D(C) = C^{-1}(e^{C} - I) \tag{20}$$

or

$$\tau D(A\tau) = A^{-1}\,(e^{A\tau} - I), \tag{21}$$

Eq. (19) becomes

$$\begin{aligned}\underline{X}(\tau) &= \underline{X}(0) + \tau A\, D(A\tau)\, \underline{X}\,(0) + \tau\, D(A\tau)\underline{s} \\ &= \underline{X}(0) + \tau\, D(A\tau)\, \left[A\underline{X}(0) + \underline{s}\right],\end{aligned} \tag{22}$$

which is Eq. (13).

In order to derive Eq. (14) we integrate Eq. (12) to obtain

$$\begin{aligned}\underline{Y}(\tau) &= B\ \left\{ A^{-1}\,(e^{A\tau} - I)\, \underline{X}(0) + \left[-A^{-1}\tau - A^{-1}A^{-1}\,(e^{A\tau} - I)\right]\underline{s} \right\} \\ &= B\ \left\{ \tau D(A\tau)\, \underline{X}(0) + \tau^{2} \left[-C^{-1} + C^{-1}\, D(C)\right]\underline{s} \right\} \\ &= B\ \left\{ \tau D(A\tau)\, \underline{X}(0) + \tau^{2}\, Z(A\tau)\underline{s} \right\},\end{aligned} \tag{23}$$

where we have defined

$$CZ(C) = D(C) - I \tag{24}$$

for $C = A\tau$ and used Eq. (21). The last line of Eq. (23) is just Eq. (14).

Note that the matrix operators D(C) and Z(C) defined by

$$D(C) = C^{-1}(e^{C} - I) = \sum_{n=0}^{\infty} \frac{C^{n}}{(n+1)!} \quad \text{and} \tag{25}$$

$$Z(C) = C^{-1}(D(C) - I) = \sum_{n=0}^{\infty} \frac{C^{n}}{(n+2)!} \tag{26}$$

exist even if $C = A\tau$ is singular.* Although the eigenvalues of e^{C} are bounded by unity, and the eigenvalues of C are bounded, but not necessarily by unity, the direct evaluation of D(C) and Z(C) would prove difficult computationally if Eqs. (25) and (26) are used. We can scale the matrix C so that the eigenvalues are bounded by unity. Define

$$H = 2^{-p}C, \tag{27}$$

where p is determined by

$$||H|| < \frac{1}{2} \tag{28}$$

or[4,6]

$$p > \ln \left(\sum_{ij}{}' |C_{ij}|^{2}\right) / (2 \ln 2). \tag{29}$$

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*For example, a chain involving a stable isotope will lead to a matrix C that is singular.

We approximate the D(H) and Z(H) matrix operators by a finite number of terms M using Eqs.(25) and (26).

$$D^M(H) \approx \sum_{n=0}^{M} \frac{H^n}{(n + 1)!} \tag{30}$$

$$Z^M(H) \approx \sum_{n=0}^{M} \frac{H^n}{(n + 2)!} \tag{31}$$

M is determined[4] such that the excluded terms have an error less than some ε, or

$$\frac{(||H||)^{M + 1}}{(M + 2)!} < \frac{1}{2^{M + 1}(M + 2)!} < \varepsilon. \tag{32}$$

Knowing D(H) and Z(H), we may recur upwards by powers of 2 in H to find D(C) and Z(C) where $C = 2^pH$, using the recursion relations

$$D(2^{p + 1}H) = D(2^pH) \left[I + \frac{1}{2} (2^pH)D(2^pH)\right] \text{ and} \tag{33}$$

$$Z(2^{p + 1}H) = \frac{1}{2} Z(2^pH) + \frac{1}{4} \left[D(2^pH)\right]^2. \tag{34}$$

These recursion relationships are proved in Appendix A.

Using the above equations, we wrote and debugged a computer program called LEAF. The LEAF program listing is given in Appendix B.

We next discuss the program logic of LEAF, the input structure, and then we examine some of the comparisons that were made for the validation of the LEAF program.

IV. LEAF PROGRAM LOGIC

The LEAF program consists of a driver routine which controls the program flow (LEAF); nine primary subroutines (INPA, INPC, MAKEA, SOLVER, FSOLVE, PREP, PAPER, TERM, and PRMAT), which perform input/output tasks; and five secondary subroutines (SCALAR, MULTI, EQUAL, MVMUL, VADD), which are called by the primary subroutines to evaluate matrix and vector operations in double precision.

The dimensions of arrays are set by a parameter statement. The meaning and the current values of the four parameters in LEAF are

NNT = 10 : Maximum number of nuclides allowed a problem,
NN = 2*NNT: Twice NNT,
NIT = 25 : Maximum number of time intervals plus one, and
NBR = 10 : Maximum number of branching ratios allowed.

These parameters can be increased as long as NN = 2*NNT. No use is made of Large Core Memory in this version of the code. The code runs using both CROS-CDC-7600 BATCH mode and NOS-CDC-6600 time-sharing terminal mode.

The nine primary subroutines are discussed in the order in which they are called by the driver routine LEAF.

A. INPA

The subroutine INPA reads and prints the basic nuclear data used in constructing the decay chain matrices. The input is stored so that it may be recalled in subsequent subroutines. The printing of this data is controlled by the value of NSKIP. If NSKIP is greater than zero, the input read by INPA is not printed.

B. INPC

The subroutine INPC reads and prints the time-dependent data, the initial concentrations, and the time-dependent source data. The initial concentrations and time-dependent sources are input in atoms and atoms/s if IAC = 0. If IAC > 0, the input is in Ci and Ci/s for radioactive isotopes ($\lambda \neq 0$), and in g and g/s for stable isotopes ($\lambda = 0$). As in INPA, the printing of the input is controlled by the value of NSKIP.

At this point all of the required inputs have been read and stored.

C. MAKEA

The subroutine MAKEA constructs the main solution matrix A defined in Eq. (7). The size of the A matrix is NN by NN. The upper half of the A matrix models the behavior of the nuclides in the containment building. The lower half of the A matrix models the behavior in the containment building filter system.

The subroutine MAKEA also constructs the matrix B, Eq. (11). B is called BL in MAKEA, and premultiplies the integrated containment concentration vector to calculate the integrated release to the environment.

D. SOLVER

The subroutine SOLVER uses the matrix A to calculate three matrix operators: $D(A\tau)$, $I + A\tau D(A\tau)$, and $Z(A\tau)$ as used in Eqs. (13) and (14). The value of p is determined using Eq. (29) to insure that $||H|| < \frac{1}{2}$, where $H = 2^{-p}C$ and $C = A\tau$. Then the power series representations for D(H) and Z(H) are evaluated, Eqs. (30) and (31). Finally, the recursion relations, Eqs. (33) and (34), are used to determine the D(C), I + CD(C) and Z(C) matrix operators needed by FSOLVE to establish the solution for a given time interval.

E. FSOLVE

The subroutine FSOLVE calculates the concentrations of the nuclides in the containment building and filter as well as the integrated release to the environment for the specified time interval, Eqs. (13) and (14), using the matrix operators determined in SOLVER.

The system of subroutines (MAKEA, SOLVER, and FSOLVE) is repeatedly evaluated for each time interval as specified by the input read by subroutine INPC.

F. PREP

Upon completion of the calculation of all the time intervals, the subroutine PREP is used to prepare the results for final output display. This subroutine converts the calculated results in atoms into curies for radioactive nuclides and into grams for stable nuclides.

G. PAPER, TERM

The results of the LEAF calculations are printed by either the subroutine PAPER or TERM. The subroutine PAPER provides a detailed and labeled presentation of the results for each time interval in atoms and in curies or grams. The subroutine TERM produces an abbreviated output for each time interval in atoms. This routine is intended for use when the output is to be displayed on an interactive terminal and is chosen when NSKIP is greater than zero.

H. PRMAT

The subroutine PRMAT prints the matrix A for each time interval if the variable MATRIX is greater than zero. The A matrix is printed by quadrant in the trigonometric convention

$$A = \begin{pmatrix} -\Lambda & -\lambda^* \\ V & -\lambda \end{pmatrix}, \tag{35}$$

where quadrant 1 = $-\lambda^*$, quadrant 2 = $-\Lambda$, quadrant 3 = V, quadrant 4 = $-\lambda$, as defined by Eqs. (1) through (4).

Finally, there are five secondary subroutines in LEAF which perform matrix and vector operations in double precision. These routines and their functions are as follows.

1. SCALAR: Multiplies a scalar times a matrix.
2. MULTI : Multiplies two matrices.
3. EQUAL : Sets one matrix equal to another.
4. MVMUL : Multiplies a matrix times a vector.
5. VADD : Adds two vectors.

V. LEAF INPUT STRUCTURE

The input for LEAF is contained in seven cards, which are divided into three sets. The first set consists of card 0, which establishes the print options. The second set consists of cards 1 and 2 and is used to define the decay chains. The third set is composed of four cards, which define the time-dependent case data.

The specific data for each of the three sets is detailed in Table I. Note the use of negative numbers in words 2, 3, and 4 of card 1. If card 1 word 2 is negative, the nuclide is not retained by the filter; for example, a noble gas. If words 3 and/or 4 of card 1 are negative, then one or two branching ratio cards, card 2, must follow the card 1 on which the negative values appeared. It should also be noted that cards 5 and 6 are entered as pairs for each time interval.

Finally, we remark that the parameter IAC in word 3 card 3 controls the units used on the input of the initial concentrations and source terms.

VI. COMPARISONS

Extensive testing of the D(C) and Z(C) algorithms was performed and compared with analytic solutions to validate the programming. Problems involving off-diagonal elements above and below the diagonal, as well as a constant times the identity matrix, were solved successfully.

Finally, as an independent test of the LEAF model equation solutions, several problems were solved analytically using a Laplace transform technique on MACSYMA.[7] We report here three such tests. These test problems are not intended to represent a real accident sequence; they were designed to test the accuracy of the LEAF solutions when compared to independently constructed analytic solutions.

The first two problems use the simple decay chain defined by[8]

$$ {}^{88}\text{Br} \longrightarrow {}^{88}\text{Kr} \longrightarrow {}^{88}\text{Rb}. \tag{36} $$

The basic data involved is given in Table II.

In these first two sample problems the source was held constant in time, either zero or a fixed value. The filtration rate and leakage rate were held constant during the course of the problem:

V = filtration rate = $2.5 \times 10^{-4} s^{-1}$

$\overline{L}$ = leakage rate = $1.157 \times 10^{-8} s^{-1}$.

The containment building inventory, filter inventory, and the integrated release were evaluated at 0, 2, 4, 6, 8, and 24 h.

TABLE I

LEAF INPUT CARDS

CARD	WORD	FORMAT	SYMBOL	DESCRIPTION
0	1	I4	NSKIP	NSKIP = 0: Unabridged output NSKIP = 1: Abbreviated terminal output
	2	I4	MATRIX	MATRIX = 0: Do not print A matrix MATRIX = 1: Print A matrix
1	1	A7	HANMAT(I, 1)	Alphanumeric nuclide name
	2	F4	HANMAT(I, 2)	Nuclide ID No., negative if nuclide not retained by filter
	3	F4	HANMAT(I, 3)	Decay Parent No. 1, negative if branching ratio involved
	4	F4	HANMAT(I, 4)	Decay Parent No. 2, negative if branching ratio involved
	5	E 12.5	ANMAT(I, 1)	Nuclear decay constant (s^{-1})
	6	E 12.5	ANMAT(I, 2)	Atomic mass in gram-atoms
2	1	E 12.5	BRV(M)	Branching ratio assoicated with first negative decay parent, if applicable
2'	1	E 12.5	BRV(M + 1)	Branching ratio associated with second negative decay parent, if applicable
A BLANK CARD MUST FOLLOW THE LAST PAIR OF CARDS 1 AND 2				
3	1	I4	INT	Number of time intervals
	2	I4	ITP	PRINTING FREQUENCY ITP = 1: Print every interval ITP = 2: Print every second interval ITP = N: Print every Nth interval
	3	I4	IAC	Input units of initial concentration and source terms IAC = 0: Atoms and atoms/s IAC = 1: Curies and curies/s if radioactive, grams if stable
	4	E 12.5	TEND(INT + 1)	Time at end of problem in hours
4	1	E 12.5	CONTIC(1)	Initial concentration (atoms) of nuclide 1 in the containment building
	2	E 12.5	CONTIC(2)	Initial concentration (atoms) of nuclide 2 in the containment building
	⋮	⋮	⋮	⋮
	I	E 12.5	CONTIC(I)	Initial concentration (atoms) of nuclide I in the containment building
5	1	E 12.5	TEND(N)	Beginning time in hours of the Nth time step
	2	E 12.5	LAMDAV(N)	Clean-up rate (s^{-1}) for Nth time step
	3	E 12.5	LAMDAL(N)	Leakage rate (s^{-1}) for Nth time step
6	1	E 12.5	SOURCE(1,N)	Source term (atoms/s) for nuclide 1 in Nth time step
	2	E 12.5	SOURCE(2,N)	Source term (atoms/s) for nuclide 2 in Nth time step
	⋮	⋮	⋮	⋮
	I	E 12.5	SOURCE(I, N)	Source term (atoms/s) for nuclide I in Nth time step
CARDS 5 AND 6 ARE ENTERED AS A PAIR				

TABLE II

BASIC DATA FOR LEAF
Tests 1 and 2

NUCLIDE	DECAY CONSTANT (s^{-1})	CONTAINMENT BUILDING CONCENTRATION At T = 0 (atoms)	SOURCE (atoms/s)	
			Test 1	Test 2
^{88}Br	4.359×10^{-2}	1.912×10^{13}	0	1×10^{18}
^{88}Kr	6.876×10^{-5}	1.090×10^{18}	0	2×10^{18}
^{88}Rb	6.527×10^{-4}	1.213×10^{14}	0	3×10^{18}

In order to verify the LEAF results, the same two test problems were solved analytically using MACSYMA.[7] The problem solved by LEAF and on MACSYMA is defined as follows.

$$\frac{d\underline{N}}{dt} = A\underline{N} + \underline{S} \, , \tag{37}$$

where

$$\underline{N} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} , \quad \underline{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} , \quad \underline{N}_o = \begin{bmatrix} 1.912 \times 10^{13} \\ 1.090 \times 10^{18} \\ 1.213 \times 10^{14} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

and

$$\underline{R}(\tau) = \bar{L} \int_0^{\tau} dt' \underline{N}(t') \tag{39}$$

with

$$R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{L} = \begin{bmatrix} 1.157 \times 10^{-8} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.157 \times 10{-8} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.157 \times 10^{-8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad (40)$$

and the A matrix given by

$$A = \begin{bmatrix} -4.384 \times 10^{-2} & 0 & 0 & 0 & 0 & 0 \\ 4.359 \times 10^{-2} & -6.8772 \times 10^{-5} & 0 & 4.359 \times 10^{-2} & 0 & 0 \\ 0 & -6.8760 \times 10^{-5} & -9.02712 \times 10^{-4} & 0 & 0 & 0 \\ 2.500 \times 10^{-4} & 0 & 0 & -4.359 \times 10^{-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.876 \times 10^{-5} & 0 \\ 0 & 0 & 2.5 \times 10^{-4} & 0 & 6.876 \times 10^{-5} & -6.527 \times 10^{-4} \end{bmatrix}. \qquad (41)$$

Comparisons of the LEAF and MACSYMA solutions are given in Tables III and IV. We note that in most instances six-digit agreement occurs, with the maximum discrepancy in the fifth digit.

The third test problem was the mass-85 chain defined by[8]

$$\begin{tikzcd} {}^{85}\mathrm{As} \arrow[r] & {}^{85}\mathrm{Se} \arrow[dr] & & & \\ & & {}^{85}\mathrm{Br} \arrow[r] & {}^{85m}\mathrm{Kr} \arrow[r] \arrow[d] & {}^{85}\mathrm{Rb}. \\ & {}^{85m}\mathrm{Se} \arrow[ur] & & {}^{85}\mathrm{Kr} \arrow[ur] & \end{tikzcd} \qquad (42)$$

As an illustration of the LEAF program, the problem cards for Eq. (42) are listed in Table V for a BATCH CROS-CDC-7600 run. In Table VI the decay chain[8] and nuclide data LEAF output are displayed.

Table VII lists the filtration removal rate, leakage removal rate and source terms for the various time intervals. Note in Table VII that a source to the containment building was non zero for all times. At 42 hours it was changed and again held constant.

Table VIII displays the A matrix constructed from the input (see Eq. 35) in quadrant form. Finally in Table IX the fission product inventories are given at t = 0, 2, 4, 6, 8, 24, 30, 36, 42, 48, 54, and 60 hours. Note in Table IX that ^{85}Rb is stable and the listing is marked with an * indicating that the inventory is given in atoms and grams, <u>not</u> atoms and curies.

TABLE III

Test Problem 1
88 CHAIN - ZERO SOURCE
(All results in atoms)

TIME (hours)	NUCLIDE	CONTAINMENT INVENTORY		FILTER INVENTORY		INTEGRATED RELEASE	
		LEAF	MACSYMA	LEAF	MACSYMA	LEAF	MACSYMA
0	^{88}Br	1.9120×10^{13}	1.9120×10^{13}	0	0	0	0
	^{88}Kr	1.0900×10^{18}	1.0900×10^{18}	0	0	0	0
	^{88}Rb	1.2130×10^{14}	1.2130×10^{14}	0	0	0	0
2	^{88}Br	0	0	3.77076×10^{-18}	0	5.04604×10^{6}	5.04604×10^{6}
	^{88}Kr	6.64341×10^{17}	6.64339×10^{17}	0	0	7.16153×10^{13}	7.16152×10^{13}
	^{88}Rb	5.46412×10^{16}	5.46410×10^{16}	2.24197×10^{16}	2.24196×10^{16}	4.75620×10^{12}	4.75619×10^{12}
4	^{88}Br	0	0	0	0	5.04604×10^{6}	5.04604×10^{6}
	^{88}Kr	4.04900×10^{17}	4.04897×10^{17}	0	0	1.15263×10^{14}	1.15263×10^{14}
	^{88}Rb	3.33846×10^{16}	3.33844×10^{16}	1.42828×10^{16}	1.42827×10^{16}	8.35332×10^{12}	8.35329×10^{12}
6	^{88}Br	0	0	0	0	5.04604×10^{6}	5.04604×10^{6}
	^{88}Kr	2.46777×10^{17}	2.46774×10^{17}	0	0	1.41865×10^{14}	1.41865×10^{14}
	^{88}Rb	2.03472×10^{16}	2.03470×10^{16}	8.71126×10^{15}	8.71119×10^{15}	1.05467×10^{13}	1.05467×10^{13}
8	^{88}Br	0	0	0	0	5.04604×10^{6}	5.04604×10^{6}
	^{88}Kr	1.50405×10^{17}	1.50403×10^{17}	0	0	1.58079×10^{14}	1.58078×10^{14}
	^{88}Rb	1.24012×10^{16}	1.24010×10^{16}	5.30937×10^{15}	5.30930×10^{15}	1.18836×10^{13}	1.18835×10^{14}
24	^{88}Br	0	0	0	0	5.04604×10^{6}	5.04604×10^{6}
	^{88}Kr	2.86362×10^{15}	2.86351×10^{15}	0	0	1.82901×10^{14}	1.82900×10^{14}
	^{88}Rb	2.36111×10^{14}	2.36102×10^{14}	1.01087×10^{14}	1.01084×10^{14}	1.39302×10^{13}	1.39301×10^{13}

TABLE IV

Test Problem 2
88 CHAIN - NON ZERO
SOURCES
(All results in atoms)

TIME (hours)	NUCLIDE	CONTAINMENT INVENTORY		FILTER INVENTORY		INTEGRATED RELEASE	
		LEAF	MACSYMA	LEAF	MACSYMA	LEAF	MACSYMA
0	$^{88}\mathrm{BR}$	1.91200×10^{13}	1.91200×10^{13}	0	0	0	0
	$^{88}\mathrm{Kr}$	1.09000×10^{18}	1.09000×10^{18}	0	0	0	0
	$^{88}\mathrm{Rb}$	1.21300×10^{14}	1.21300×10^{14}	0	0	0	0
2	$^{88}\mathrm{Br}$	2.28102×10^{19}	2.28102×10^{19}	1.30823×10^{17}	1.30822×10^{17}	1.89416×10^{15}	1.89416×10^{15}
	$^{88}\mathrm{Kr}$	1.70224×10^{22}	1.70223×10^{22}	0	0	7.66459×10^{17}	7.66458×10^{17}
	$^{88}\mathrm{Rb}$	4.44825×10^{21}	4.44824×10^{21}	1.57431×10^{21}	1.57431×10^{21}	2.78215×10^{17}	2.78214×10^{17}
4	$^{88}\mathrm{Br}$	2.28102×10^{19}	2.28102×10^{19}	1.30823×10^{17}	1.30822×10^{17}	3.79434×10^{15}	3.79434×10^{15}
	$^{88}\mathrm{Kr}$	2.74104×10^{22}	2.74103×10^{22}	0	0	2.65273×10^{18}	2.65273×10^{18}
	$^{88}\mathrm{Rb}$	5.30934×10^{21}	5.30933×10^{21}	1.97298×10^{21}	1.97298×10^{21}	6.87702×10^{17}	6.87702×10^{17}
6	$^{88}\mathrm{Br}$	2.28102×10^{19}	2.28102×10^{19}	1.30823×10^{17}	1.30822×10^{17}	5.69453×10^{15}	5.69453×10^{15}
	$^{88}\mathrm{Kr}$	3.37417×10^{22}	3.37416×10^{22}	0	0	5.22152×10^{18}	5.22150×10^{18}
	$^{88}\mathrm{Rb}$	5.83137×10^{21}	5.83136×10^{21}	2.19680×10^{21}	2.19680×10^{21}	1.15352×10^{18}	1.15352×10^{18}
8	$^{88}\mathrm{Br}$	2.28102×10^{19}	2.28102×10^{19}	1.30823×10^{17}	1.30822×10^{17}	7.59471×10^{15}	7.59471×10^{15}
	$^{88}\mathrm{Kr}$	3.76004×10^{22}	3.76003×10^{22}	0	0	8.20627×10^{18}	8.20624×10^{18}
	$^{88}\mathrm{Rb}$	6.14954×10^{21}	6.14952×10^{21}	2.33302×10^{21}	2.33302×10^{21}	1.65364×10^{18}	1.65364×10^{18}
24	$^{88}\mathrm{Br}$	2.28102×10^{19}	2.28102×10^{19}	1.30823×10^{17}	1.30822×10^{17}	2.27962×10^{16}	2.27962×10^{16}
	$^{88}\mathrm{Kr}$	4.35080×10^{22}	4.35077×10^{22}	0	0	3.62839×10^{19}	3.62838×10^{19}
	$^{88}\mathrm{Rb}$	6.63663×10^{21}	6.63660×10^{21}	2.54156×10^{21}	2.54155×10^{21}	6.00086×10^{18}	6.00084×10^{18}

TABLE V

LIST OF TEST PROBLEM 3 DECK FOR BATCH CROS CDC-7600

```
$JOB(NAME=YSAPP,AC=ADOT ,CL=U,UA=9R04R401,PR=22,FL=50,TL=30S)
$REWIND(JOBIN)
$COPYSF(I=JOBIN,O=DUM,N=3)
$AFSREL(FS=DUM,ADISP=PRT)
$CLOSER(FS=JOBIN)
$OPERM(FS=MACLIB2,OAC=ZDO,FSI=ZFSI)
$MACRO(MDF=MACLIB2,OAC=ZDO)
$PHOTOR(FS=LEAF)
$UPDATE(F,P=LEAF)
$RUN(I=COMPILE)
$LDGO(SETA=I)
$FM.
*COMPILE,LEAF
$FM.
   0    1
AS 85       1    0    0 3.415   E-01 85.0
SE 85       2   -1    0 1.777   E-02 85.0
 .800
SE 85M      3    0    0 3.648   E-02 85.0
BR 85       4    2    3 4.025   E-03 85.0
KR 85M     -5    4    0 4.298   E-05 85.0
KR 85      -6   -5    0 2.047   E-09 85.0
 .212
RB 85       7   -5    6 0.0          85.0
 .788

   11    1    0 60.0


 0.0            2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 2.0            2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 4.0            2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 6.0            2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 8.0            2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 24.0           2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 30.0           2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 36.0           2.5       E-04 1.157   E-08
 1.0       E+18 1.5       E+18 2.0     E+18 2.5      E+18 3.0      E+18 3.5      E+18
 4.0       E+18
 42.0           2.5       E-04 1.157   E-08
 4.0       E+18 3.5       E+18 3.0     E+18 2.5      E+18 2.0      E+18 1.0      E+18
 1.0       E+18
 48.0           2.5       E-04 1.157   E-08
 4.0       E+18 3.5       E+18 3.0     E+18 2.5      E+18 2.0      E+18 1.0      E+18
 1.0       E+18
 54.0           2.5       E-04 1.157   E-08
 4.0       E+18 3.5       E+18 3.0     E+18 2.5      E+18 2.0      E+18 1.0      E+18
 1.0       E+18
$EJ.
```

TABLE VI

DECAY CHAINS AND NUCLIDE RELATED DATA

NUCLIDE	ID	PARENT 1	PARENT 2	DECAY CONSTANT	ATOMIC MASS
AS 85	1	0	0	3.41500D-01	8.50000D+01
SE 85	2	-1	0	1.77700D-02	8.50000D+01
SE 85M	3	0	0	3.64800D-02	8.50000D+01
BR 85	4	2	3	4.02500D-03	8.50000D+01
KR 85M	-5	4	0	4.29800D-05	8.50000D+01
KR 85	-6	-5	0	2.04700D-09	8.50000D+01
RB 85	7	-5	6	0.	8.50000D+01

BRANCHING RATIO

FROM	TO	RATIO
1	2	.8000
5	6	.2120
5	7	.7880

TABLE VII

FILTRATION REMOVAL RATE, LEAKAGE REMOVAL RATE, AND SOURCE TERMS FOR TIME INTERVALS

```
                                              TIME STEP DATA

                                          TIME PERIOD IN HOURS

                           0.0- 2.0     2.0- 4.0     4.0- 6.0     6.0- 8.0     8.0- 24.0    24.0- 30.0   30.0- 36.0

FILTRATION REMOVAL RATE   2.50000D-04  2.50000D-04  2.50000D-04  2.50000D-04  2.50000D-04  2.50000D-04  2.50000D-04

LEAKAGE REMOVAL RATE      1.15700D-08  1.15700D-08  1.15700D-08  1.15700D-08  1.15700D-08  1.15700D-08  1.15700D-08

SOURCE TERMS
  AS 85                   1.00000D+18  1.00000D+18  1.00000D+18  1.00000D+18  1.00000D+18  1.00000D+18  1.00000D+18
  SE 85                   1.50000D+18  1.50000D+18  1.50000D+18  1.50000D+18  1.50000D+18  1.50000D+18  1.50000D+18
  SE 85M                  2.00000D+18  2.00000D+18  2.00000D+18  2.00000D+18  2.00000D+18  2.00000D+18  2.00000D+18
  BR 85                   2.50000D+18  2.50000D+18  2.50000D+18  2.50000D+18  2.50000D+18  2.50000D+18  2.50000D+18
  KR 85M                  3.00000D+18  3.00000D+18  3.00000D+18  3.00000D+18  3.00000D+18  3.00000D+18  3.00000D+18
  KR 85                   3.50000D+18  3.50000D+18  3.50000D+18  3.50000D+18  3.50000D+18  3.50000D+18  3.50000D+18
  RB 85                   4.00000D+18  4.00000D+18  4.00000D+18  4.00000D+18  4.00000D+18  4.00000D+18  4.00000D+18


                          36.0- 42.0   42.0- 48.0   48.0- 54.0   54.0- 60.0

FILTRATION REMOVAL RATE   2.50000D-04  2.50000D-04  2.50000D-04  2.50000D-04

LEAKAGE REMOVAL RATE      1.15700D-08  1.15700D-08  1.15700D-08  1.15700D-08

SOURCE TERMS
  AS 85                   1.00000D+18  4.00000D+18  4.00000D+18  4.00000D+18
  SE 85                   1.50000D+18  3.50000D+18  3.50000D+18  3.50000D+18
  SE 85M                  2.00000D+18  3.00000D+18  3.00000D+18  3.00000D+18
  BR 85                   2.50000D+18  2.50000D+18  2.50000D+18  2.50000D+18
  KR 85M                  3.00000D+18  2.00000D+18  2.00000D+18  2.00000D+18
  KR 85                   3.50000D+18  1.00000D+18  1.00000D+18  1.00000D+18
  RB 85                   4.00000D+18  1.00000D+18  1.00000D+18  1.00000D+18
```

TABLE VIII

THE A MATRIX PRINTED BY QUADRANTS

```
  QUADRANT  1

 0.            0.            0.            0.            0.            0.            0.
 0.            0.            0.            0.            0.            0.            0.
 0.            0.            0.            0.            0.            0.            0.
 0.            0.            0.            0.            0.            0.            0.
 0.            0.            0.            4.02500D-03   0.            0.            0.
 0.            0.            0.            0.            9.11176D-06   0.            0.
 0.            0.            0.            0.            0.            0.            0.
  QUADRANT  2

-3.41750D-01   0.            0.            0.            0.            0.            0.
 2.73200D-01  -1.80200D-02   0.            0.            0.            0.            0.
 0.            0.           -3.67300D-02   0.            0.            0.            0.
 0.            1.77700D-02   3.64800D-02  -4.27501D-03   0.            0.            0.
 0.            0.            0.            4.02500D-03  -4.29916D-05   0.            0.
 0.            0.            0.            0.            9.11176D-06  -1.36170D-08   0.
 0.            0.            0.            0.            3.38682D-05   2.04700D-09  -2.50012D-04
  QUADRANT  3

 2.50000D-04   0.            0.            0.            0.            0.            0.
 0.            2.50000D-04   0.            0.            0.            0.            0.
 0.            0.            2.50000D-04   0.            0.            0.            0.
 0.            0.            0.            2.50000D-04   0.            0.            0.
 0.            0.            0.            0.            0.            0.            0.
 0.            0.            0.            0.            0.            0.            0.
 0.            0.            0.            0.            0.            0.            2.50000D-04
  QUADRANT  4

-3.41500D-01   0.            0.            0.            0.            0.            0.
 2.73200D-01  -1.77700D-02   0.            0.            0.            0.            0.
 0.            0.           -3.64800D-02   0.            0.            0.            0.
 0.            1.77700D-02   3.64800D-02  -4.02500D-03   0.            0.            0.
 0.            0.            0.            0.           -4.29800D-05   0.            0.
 0.            0.            0.            0.            0.           -2.04700D-09   0.
 0.            0.            0.            0.            3.38682D-05   2.04700D-09  -0.
```

TABLE IX
FISSION PRODUCT INVENTORIES

```
                         FISSION PRODUCT INVENTORY AT    0.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

   NUCLIDE        CONTAINMENT INVENTORY          FILTER INVENTORY            INTEGRATED RELEASE

                     ATOMS      CURIES,GM       ATOMS      CURIES,GM        ATOMS      CURIES,GM

   AS 85          -0.          0.              0.         0.               0.         0.
   SE 85          -0.          0.              0.         0.               0.         0.
   SE 85M         -0.          0.              0.         0.               0.         0.
   BR 85          -0.          0.              0.         0.               0.         0.
   KR 85M         -0.          0.              0.         0.               0.         0.
   KR 85          -0.          0.              0.         0.               0.         0.
 * RB 85          -0.          0.              0.         0.               0.         0.

                         FISSION PRODUCT INVENTORY AT    2.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

   NUCLIDE        CONTAINMENT INVENTORY          FILTER INVENTORY            INTEGRATED RELEASE

                     ATOMS      CURIES,GM       ATOMS      CURIES,GM        ATOMS      CURIES,GM

   AS 85          2.92612D+18  2.70073D+07    2.14211D+15  1.97711D+04    2.43658D+14  2.24890D+03
   SE 85          1.27603D+20  6.12841D+07    1.82814D+18  8.78002D+05    1.05464D+16  5.06514D+03
   SE 85M         5.44514D+19  5.36861D+07    3.73159D+17  3.67915D+05    4.51887D+15  4.45536D+03
   BR 85          1.57986D+21  1.71863D+08    1.09581D+20  1.19206D+07    1.26839D+17  1.37981D+04
   KR 85M         5.92923D+22  6.88752D+07    0.           0.             2.52747D+18  2.93597D+03
   KR 85          2.71892D+22  1.50422D+03    0.           0.             1.10485D+18  6.11250D-02
 * RB 85          1.77564D+22  2.50609D+00    1.84415D+22  2.60278D+00    8.53471D+17  1.20457D-04
```

```
                         FISSION PRODUCT INVENTORY AT    4.00 HOURS

         STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE        CONTAINMENT INVENTORY           FILTER INVENTORY              INTEGRATED RELEASE

                  ATOMS        CURIES,GM        ATOMS        CURIES,GM        ATOMS        CURIES,GM

  AS 85        2.92612D+18  2.70073D+07      2.14211D+15  1.97711D+04      4.87415D+14  4.49871D+03
  SE 85        1.27603D+20  6.12841D+07      1.82814D+18  8.78002D+05      2.11763D+16  1.01704D+04
  SE 85M       5.44514D+19  5.36861D+07      3.73159D+17  3.67915D+05      9.05488D+15  8.92763D+03
  BR 85        1.57986D+21  1.71863D+08      1.09581D+20  1.19206D+07      2.58448D+17  2.81149D+04
  KR 85M       1.04192D+23  1.21031D+08      0.           0.               9.43324D+18  1.09579D+04
  KR 85        5.78235D+22  3.19905D+03      0.           0.               4.62540D+18  2.55897D-01
* RB 85        2.63796D+22  3.72314D+00      5.88320D+22  8.30339D+00      2.72274D+18  3.84281D-04


                         FISSION PRODUCT INVENTORY AT    6.00 HOURS

         STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE        CONTAINMENT INVENTORY           FILTER INVENTORY              INTEGRATED RELEASE

                  ATOMS        CURIES,GM        ATOMS        CURIES,GM        ATOMS        CURIES,GM

  AS 85        2.92612D+18  2.70073D+07      2.14211D+15  1.97711D+04      7.31172D+14  6.74852D+03
  SE 85        1.27603D+20  6.12841D+07      1.82814D+18  8.78002D+05      3.18062D+16  1.52756D+04
  SE 85M       5.44514D+19  5.36861D+07      3.73159D+17  3.67915D+05      1.35009D+16  1.33999D+04
  BR 85        1.57986D+21  1.71863D+08      1.09581D+20  1.19206D+07      3.90056D+17  4.24317D+04
  KR 85M       1.37138D+23  1.59303D+08      0.           0.               1.95558D+19  2.27165D+04
  KR 85        9.09881D+22  5.03386D+03      0.           0.               1.08087D+19  5.97987D-01
* RB 85        3.19810D+22  4.51371D+00      1.11660D+23  1.57595D+01      5.16765D+18  7.29348D-04
```

```
                         FISSION PRODUCT INVENTORY AT    8.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE        CONTAINMENT INVENTORY          FILTER INVENTORY           INTEGRATED RELEASE

                 ATOMS        CURIES.GM       ATOMS        CURIES.GM       ATOMS        CURIES.GM

  AS 85       2.92612D+18  2.70073D+07    2.14211D+15  1.97711D+04    9.74929D+14  8.99834D+03
  SE 85       1.27663D+20  6.12841D+07    1.82814D+18  8.78002D+05    4.24361D+16  2.03808D+04
  SE 85M      5.44514D+19  5.36861D+07    3.73159D+17  3.67915D+05    1.81269D+16  1.78722D+04
  BR 85       1.57986D+21  1.71863D+08    1.09581D+20  1.19206D+07    5.21664D+17  5.67486D+04
  KR 85M      1.61314D+23  1.87386D+08    0.           0.             3.20388D+19  3.72170D+04
  KR 85       1.26068D+23  6.97132D+03    0.           0.             1.98361D+19  1.09742D+00
* RB 85       3.59713D+22  5.07689D+00    1.73010D+23  2.44181D+01    8.00689D+18  1.13007D-03


                         FISSION PRODUCT INVENTORY AT   24.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE        CONTAINMENT INVENTORY          FILTER INVENTORY           INTEGRATED RELEASE

                 ATOMS        CURIES.GM       ATOMS        CURIES.GM       ATOMS        CURIES.GM

  AS 85       2.92612D+18  2.70073D+07    2.14211D+15  1.97711D+04    2.92499D+15  2.69968D+04
  SE 85       1.27663D+20  6.12841D+07    1.82814D+18  8.78002D+05    1.27475D+17  6.12225D+04
  SE 85M      5.44514D+19  5.36861D+07    3.73159D+17  3.67915D+05    5.44151D+16  5.36503D+04
  BR 85       1.57986D+21  1.71863D+08    1.09581D+20  1.19206D+07    1.57453D+18  1.71283D+05
  KR 85M      2.23350D+23  2.59287D+08    0.           0.             1.67527D+20  1.94603D+05
  KR 85       4.34092D+23  2.40158D+04    0.           0.             2.04864D+20  1.13339D+01
* RB 85       4.59661D+22  6.48754D+00    7.90025D+23  1.11502D+02    3.65624D+19  5.16032D-03
```

```
                         FISSION PRODUCT INVENTORY AT   30.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE          CONTAINMENT INVENTORY          FILTER INVENTORY            INTEGRATED RELEASE

                    ATOMS        CURIES,GM        ATOMS        CURIES,GM        ATOMS        CURIES,GM

  AS 85          2.92612D+18  2.70073D+07     2.14211D+15  1.97711D+04     3.65626D+15  3.37463D+04
  SE 85          1.27603D+20  6.12841D+07     1.82814D+18  8.78002D+05     1.59365D+17  7.65381D+04
  SE 85M         5.44514D+19  5.36861D+07     3.73159D+17  3.67915D+05     6.80531D+16  6.70671D+04
  BR 85          1.57986D+21  1.71863D+08     1.09581D+20  1.19206D+07     1.96935D+18  2.14234D+05
  KR 85M         2.25738D+23  2.62222D+08     0.           0.              2.23583D+20  2.59718D+05
  KR 85          5.53692D+23  3.06327D+04     0.           0.              3.28280D+20  1.81619D+01
* RB 85          4.65214D+22  6.56591D+00     1.03997D+24  1.46779D+02     4.81298D+19  6.79292D-03


                         FISSION PRODUCT INVENTORY AT   36.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE          CONTAINMENT INVENTORY          FILTER INVENTORY            INTEGRATED RELEASE

                    ATOMS        CURIES.GM        ATOMS        CURIES,GM        ATOMS        CURIES,GM

  AS 85          2.92612D+18  2.70073D+07     2.14211D+15  1.97711D+04     4.38753D+15  4.04957D+04
  SE 85          1.27603D+20  6.12841D+07     1.82814D+18  8.78002D+05     1.91254D+17  9.18538D+04
  SE 85M         5.44514D+19  5.36861D+07     3.73159D+17  3.67915D+05     8.16312D+16  8.04839D+04
  BR 85          1.57986D+21  1.71863D+08     1.09581D+20  1.19206D+07     2.36418D+18  2.57184D+05
  KR 85M         2.27077D+23  2.63777D+08     0.           0.              2.80190D+20  3.25475D+05
  KR 85          6.73692D+23  3.72716D+04     0.           0.              4.81645D+20  2.66467D+01
* RB 85          4.67413D+22  6.50695D+00     1.29187D+24  1.82331D+02     5.97877D+19  8.43828D-03
```

```
                         FISSION PRODUCT INVENTORY AT   42.00 HOURS

             STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE           CONTAINMENT INVENTORY          FILTER INVENTORY            INTEGRATED RELEASE

                     ATOMS       CURIES.GM        ATOMS       CURIES.GM        ATOMS       CURIES.GM

  AS 85           2.92612D+18  2.70073D+07     2.14211D+15  1.97711D+04     5.11880D+15  4.72451D+04
  SE 85           1.27603D+20  6.12841D+07     1.82814D+18  8.78002D+05     2.23144D+17  1.07169D+05
  SE 85M          5.44514D+19  5.36851D+07     3.73159D+17  3.67915D+05     9.52392D+16  9.39007D+04
  BR 85           1.57986D+21  1.71863D+08     1.09581D+20  1.19206D+07     2.75900D+18  3.00135D+05
  KR 85M          2.27606D+23. 2.64392D+08     0.           0.              3.37015D+20  3.91484D+05
  KR 85           7.03828D+23  4.39180D+04     0.           0.              6.65019D+20  3.67917D+01
* RB 85           4.48289D+22  6.60930D+00     1.54454D+24  2.17993D+02     7.14815D+19  1.00887D-02

                         FISSION PRODUCT INVENTORY AT   48.00 HOURS

             STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE           CONTAINMENT INVENTORY          FILTER INVENTORY            INTEGRATED RELEASE

                     ATOMS       CURIES.GM        ATOMS       CURIES.GM        ATOMS       CURIES.GM

  AS 85           1.17045D+19  1.08029D+08     8.56842D+15  7.90842D+04     8.04359D+15  7.42402D+04
  SE 85           3.71679D+20  1.78506D+08     5.36076D+18  2.57461D+06     3.15870D+17  1.51703D+05
  SE 85M          8.16771D+19  8.05292D+07     5.59739D+17  5.51872D+05     1.15643D+17  1.14017D+05
  BR 85           2.82673D+21  3.07503D+08     2.04314D+20  2.22260D+07     3.46132D+18  3.76536D+05
  KR 85M          2.89075D+23  3.35795D+08     0.           0.              4.02581D+20  4.67647D+05
  KR 85           8.46819D+23  4.79562D+04     0.           0.              8.72272D+20  4.82579D+01
* RB 85           4.20756D+22  5.93844D+00     1.76285D+24  2.48804D+02     8.15847D+19  1.15147D-02
```

```
                    FISSION PRODUCT INVENTORY AT   54.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE           CONTAINMENT INVENTORY           FILTER INVENTORY            INTEGRATED RELEASE

                   ATOMS        CURIES,GM        ATOMS        CURIES,GM        ATOMS        CURIES,GM

  AS 85         1.17045D+19  1.08029D+08     8.56842D+15  7.90842D+04     1.09687D+16  1.01238D+05
  SE 85         3.71679D+20  1.78506D+08     5.36076D+18  2.57461D+06     4.08757D+17  1.96314D+05
  SE 85M        8.16771D+19  8.05292D+07     5.59739D+17  5.51872D+05     1.36055D+17  1.34143D+05
  BR 85         2.82673D+21  3.07503D+08     2.04314D+20  2.22260D+07     4.16776D+18  4.53384D+05
  KR 85M        3.14010D+23  3.64761D+08     0.           0.               4.78415D+20  5.55738D+05
  KR 85         9.47875D+23  5.24405D+04     0.           0.               1.09893D+21  6.07975D+01
* RB 85         4.60874D+22  6.50466D+00     2.00245D+24  2.82621D+02     9.26736D+19  1.30797D-02


                    FISSION PRODUCT INVENTORY AT   60.00 HOURS

          STABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND ARE NOTED BY A * IN THE MARGIN

  NUCLIDE           CONTAINMENT INVENTORY           FILTER INVENTORY            INTEGRATED RELEASE

                   ATOMS        CURIES,GM        ATOMS        CURIES,GM        ATOMS        CURIES,GM

  AS 85         1.17045D+19  1.08029D+08     8.56842D+15  7.90842D+04     1.38938D+16  1.28236D+05
  SE 85         3.71679D+20  1.78506D+08     5.36076D+18  2.57461D+06     5.01644D+17  2.40925D+05
  SE 85M        8.16771D+19  8.05292D+07     5.59739D+17  5.51874D+05     1.56467D+17  1.54268D+05
  BR 85         2.02673D+21  3.07503D+08     2.04314D+20  2.22260D+07     4.87419D+18  5.30233D+05
  KR 85M        3.23862D+23  3.74205D+08     0.           0.               5.58309D+20  6.48544D+05
  KR 85         1.03210D+24  5.71004D+04     0.           0.               1.34630D+21  7.44831D+01
* RB 85         4.76996D+22  6.73219D+00     2.25634D+24  3.18454D+02     1.04424D+20  1.47381D-02
```

The MACSYMA analytic solution agreed in all instances to within two digits in the sixth place. This difference is judged insignificant.

The total running time on the CDC-7600 for example 3 with LEAF was 15.3 s; the problem solution time was 5.5 s.

VII. CONCLUSIONS

An analytic solution has been obtained for a containment building model to calculate the leakage into the environment of each isotope of an arbitrary radioactive decay chain. The model accounts for the source, the buildup, the decay, the cleanup and the leakage of isotopes that are gas-borne inside the containment building.

Three assumptions were made in the model: (1) the gas inside the containment building is well mixed and all in one compartment; (2) that natural deposition of gas-borne isotopes internal to the containment building is ignored; and (3) that the source of an isotope inside the containment building which is a result of leakage from the reactor vessel, its removal rate by the containment cleanup system, and its leakage from the containment building are all assumed constant during short time intervals.

With these assumptions the model is representable by a system of linear differential equations. An analytic solution is obtained to these equations in terms of matrix operators using the Volterra method of the multiplicative integral. Recursion formulae are developed to accurately evaluate the matrix operators for arbitrary matrix element values.

A computer program LEAF was written, debugged, and described. Comparisons of LEAF with those achieved by Laplace transform techniques on MACSYMA[7] indicate that the LEAF model is accurate. Computationally LEAF is fast.

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4. C. E. Lee, "The Calculation of Isotopic Mass and Energy Production by a Matrix Operator Method," Los Alamos Scientific Laboratory report LA-6483-MS (September 1976).

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APPENDIX A

$D(2^pH)$ AND $Z(2^pH)$ RECURSION RELATIONS

We demonstrate here an induction proof of the recursion relations for Eqs. (33) and (34) of the text.

$\underline{D(2^pH)}$: Define

$$D(H) = H^{-1}(e^H - 1) \qquad \text{(A-1)}$$

and

$$C = 2^pH. \qquad \text{(A-2)}$$

Clearly if $p = 0$

$$D(C) = D(H). \qquad \text{(A-3)}$$

If $p = 1$

$$\begin{aligned} D(C) = D(2H) &= (2H)^{-1}(e^{2H} - I) \\ &= H^{-1}(e^H - I)\left(\frac{e^H + I}{2}\right) \\ &= D(H)\left[I + \frac{1}{2}HD(H)\right]. \end{aligned} \qquad \text{(A-4)}$$

By induction we may write

$$D(2^pH) = D(2^{p-1}H)\left[I + \frac{1}{2}(2^{p-1}H)D(2^{p-1}H)\right]. \qquad \text{(A-5)}$$

We assume Eq. (A-5), which is true for $p = 0$ and 1, is true for $p = n$. Evaluate $D(2^{n+1}H)$ as

$$\begin{aligned} D(2^{n+1}H) &= (2^{n+1}H)^{-1}(e^{2^{n+1}H} - I) \\ &= (2^nH)^{-1}(e^{2^nH} - I)\,\frac{1}{2}\,(e^{2^nH} + I) \\ &= D(2^nH)\left[I + \frac{1}{2}(2^nH)D(2^nH)\right]. \end{aligned} \qquad \text{(A-6)}$$

Since Eq. (A-5) is true for $p = 0$ and 1 and if it is assumed true for $p = n$, it is true for $p = n + 1$; then by transfinite induction it is true for all p.

$\underline{Z(2^pH)}$: Define

$$HZ(H) + I = D(H). \qquad \text{(A-7)}$$

Assume Eq. (A-5) is true, as was proved. Then using Eqs. (A-5) and (A-6) we may write

$$2^{p+1}H\, Z(2^{p+1}H) + I = D(2^{p+1}H)$$

$$= D(2^pH)\left[I + \frac{1}{2}(2^pH)D(2^pH)\right], \qquad \text{(A-8)}$$

$$2^pH\, Z(2^pH) + I = D(2^pH) \qquad \text{(A-9)}$$

or substituting the LHS of Eq. (A-9) into the RHS of Eq. (A-8) for $D(2^pH)$ we have

$$H\ \ Z(2^{p+1}H) = H\left\{\frac{1}{2}Z(2^pH) + \left[\frac{I}{2} + \frac{1}{2}(2^pH)Z(2^pH)\right]^2\right\} \qquad \text{(A-10)}$$

or

$$Z(2^{p+1}H) = \frac{1}{2}Z(2^pH) + \left[\frac{I}{2} + \frac{1}{2}(2^pH)Z(2^pH)\right]^2. \qquad \text{(A-11)}$$

If H is singular, we may define a non-singular matrix H' such that

$$H' = H - \varepsilon I, \quad \varepsilon << 1 \qquad \text{(A-12)}$$

and

$$|H'| \neq 0\,, \qquad \text{(A-13)}$$

which permits Eq. (A-11) to be written with H', since $(H')^{-1}$ exists and yet is arbitrarily close to H. The H matrices in LEAF will be singular if a stable isotope is in a chain.

Since Z(H) exists even if H is singular [see Eq. (26)], an alternate proof of the validity of Eq. (A-11) can be made by direct evaluation of the power series. That tedious process will not be repeated here; however, term by term comparison indicates that Eq. (A-11) is indeed correct.

Computationally, Eq. (A-11) is subject to round-off errors even in double precision arithmetic. Using Eq. (A-9) in Eq. (A-11) we can eliminate that difficulty and obtain

$$Z\ (2^{p+1}H) = \frac{1}{2}Z(2^pH) + \frac{1}{4}\left[D(2^pH)\right]^2, \qquad \text{(A-14)}$$

which involves evaluating Z(C) after D(C) at each point in the recursion.

LASL Identification Code: LP-0722

APPENDIX B: LEAF PROGRAM LISTING

```
      PROGRAM LEAF (INP,OUT)
C
C     LEAF - A COMPUTER PROGRAM TO CALCULATE FISSION PRODUCT RELEASE
C     FROM A REACTOR CONTAINMENT BUILDING FOR ARBITRARY RADIOACTIVE
C     DECAY CHAINS
C
C     BY CLARENCE E. LEE
C        COURTNEY E. APPERSON, JR.
C        JOHN E. FOLEY
C
C     REACTOR TECHNOLOGY DIVISION
C     LOS ALAMOS SCIENTIFIC LABORATORY
C     NOVEMBER 1976
C
C    THIS IS THE EIGHTH VERSION OF LEAF.  IT WAS CREATED ON
C    11 NOVEMBER 1976.  THE CHANGES INCORPORATED IN VERSION
C    EIGHT REMOVE A DIMENSION ERROR.
C
C  INPUT INSTRUCTIONS FOR THE CODE FOLLOW
C
C  CARD   WORD     SYMBOL     FORMAT   INFORMATION
C    0  PRINT OPTION
C           1       NSKIP       I4     0/1 LINE PRINTER/TERMINAL
C           2       MATRIX      I4     0/1 PRINT A MATRIX NO/YES
C    1  NUCLIDE BASIC DATA - ONE CARD PER NUCLIDE
C           1   HANMAT(I,1)     A7     NUCLIDE NAME
C           2   HANMAT(I,2)     F4     ID NUMBER (NEGATIVE IF NOT
C                                      RETAINED BY FILTER)
C           3   HANMAT(I,3)     F4     DECAY PARENT 1
C           4   HANMAT(I,4)     F4     DECAY PARENT 2
C           5    ANMAT(I,1)   E12.5    NUCLEAR DECAY CONSTANT
C           6    ANMAT(I,2)   E12.5    ATOMIC MASS
C    2  BRANCHING RATIO - ONE CARD PER BRANCH AFTER CARD ONE
C       FOR EACH NEGATIVE DECAY PARENT
C           1       BRV(IBR)  E12.5    BRANCHING RATIO
C  BLANK CARD AFTER LAST SET OF CARDS ONE AND TWO
C
C    3  NUMBER OF TIME STEPS
C           1       INT         I4     NUMBER OF TIME INTERVALS
C           2       ITP         I4     PRINT FREQUENCY
C           3       IAC         I4     UNITS OF CONTIC AND SOURCE
C                                      0 - ATOMS AND ATOMS/SEC
C                                      1 - CURIES OR GRAMS AND
C                                          CURIES/SEC OR GRAMS/SEC
C           4   TEND(INT+1)   E12.5    MAXIMUM TIME IN HOURS
C    4  INITIAL VALUE OF ALL NUCLIDE CONCENTRATIONS IN CONTAINMENT
C           1    CONTIC(1)    E12.5    INITIAL CONCENTRATION OF NUCLIDE
C           2    CONTIC(2)    E12.5    INITIAL CONCENTRATION OF NUCLIDE
C           N    CONTIC(N)    E12.5    INITIAL CONCENTRATION OF NUCLIDE
C    5  TIME STEP DATA - INT CARDS
C           1      TEND(N)    E12.5    BEGINNING OF NTH TIME STEP
C                                      IN HOURS
C           2     LAMDAV(N)   E12.5    CLEAN UP RATE DURING INTERVAL
C           3     LAMDAL(N)   E12.5    LEAKAGE RATE DURING INTERVAL
C    6  TIME STEP SOURCE DATA - CARDS FIVE AND SIX ARE ENTERED AS A SET
C           1   SOURCE(1,INT) E12.5    SOURCE TERM FOR NUCLIDE 1
C           2   SOURCE(2,INT) E12.5    SOURCE TERM FOR NUCLIDE 2
C           N   SOURCE(N,INT) E12.5    SOURCE TERM FOR NUCLIDE N
C
C  PROGRAM LOGIC OF LEAF
```

```
C
C      INPA
C      INPC
C      MAKEA
C      SOLVER
C      FSOLVE
C      PREP
C      PAPER OR TERM
C
C
C    NN=20                   NUMBER OF NUCLIDES TIMES TWO
C    NNT=10                  NUMBER OF NUCLIDES
C    NIT=25                  NUMBER OF TIME INTERVALS PLUS ONE
C    NBR=10                  NUMBER OF BRANCHING RATIOS
C
      IMPLICIT DOUBLE(A-G,P-Z)
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)
      COMMON /BASIS1/ I,IBR,NSKIP
      DOUBLE LAMDV,LAMDL
      DOUBLE LAMDAV(NIT),LAMDAL(NIT)
      DIMENSION A(NN,NN),SOURCE(NNT,NIT),CONTIC(NN),XNTOT(NN),
     1 XOUT(NNT,6,NIT),XIENV(NN),TEND(NIT),ANMAT(NNP,2),BRV(NBR),
     2 B(NN,NN),D(NN,NN),E(NN,NN),BL(NN,NN),CSOURC(NN),HANMAT(NNP,4),
     3 XENVIC(NN)
C
C *** READ LEAF INPUT DATA
C *** NSKIP GREATER THAN ZERO - DO NOT PRINT INPUT DATA AND USE
C *** TERMINAL OUTPUT FORMAT
      READ 90, NSKIP,MATRIX
      CALL INPA(ANMAT,HANMAT,BRV)
      I2=I*2
      DO 10 IK=1,I2
      XNTOT(IK)=0.0D0
      XIENV(IK)=0.0D0
      XENVIC(IK)=0.0D0
      CSOURC(IK)=0.0D0
      CONTIC(IK)=0.0D0
   10 CONTINUE
      CALL INPC(INT,CONTIC,TEND,LAMDAV,LAMDAL,SOURCE,HANMAT,ITP,ANMAT)
      DO 20 J=1,I
      XOUT(J,1,1)=CONTIC(J)
      XOUT(J,3,1)=0.0D0
      XOUT(J,5,1)=0.0D0
   20 CONTINUE
      DO 50 IT=1,INT
      TINCD=(TEND(IT+1)-TEND(IT))*3600.0D0
      DO 30 IN=1,I
      CSOURC(IN)=SOURCE(IN,IT)
   30 CONTINUE
      LAMDV=LAMDAV(IT)
      LAMDL=LAMDAL(IT)
      CALL MAKEA(ANMAT,HANMAT,BRV,A,BL,LAMDV,LAMDL)
      IF(MATRIX.EQ.1) CALL PRMAT(A,IT)
      CALL SOLVER(A,B,D,E,TINCD)
      CALL FSOLVE(BL,B,D,E,CONTIC,CSOURC,XNTOT,XIENV,TINCD,XENVIC)
      DO 40 J=1,I
      XOUT(J,1,IT+1)=XNTOT(J)
      XOUT(J,3,IT+1)=XNTOT(J+I)
      XOUT(J,5,IT+1)=XIENV(J)
   40 CONTINUE
   50 CONTINUE
      INT=INT+1
      CALL PREP(XOUT,ANMAT,INT)
```

```
      IF (NSKIP) 60,60,70                                               LEAF     122
   60 CALL PAPER(XOUT,ANMAT,HANMAT,INT,ITP,TEND)                        LEAF     123
      GO TO 80                                                          LEAF     124
   70 CALL TERM(XOUT,HANMAT,INT,ITP,TEND)                               LEAF     125
   80 CONTINUE                                                          LEAF     126
C                                                                       LEAF     127
   90 FORMAT(2I4)                                                       LEAF     128
      END                                                               LEAF     129
      SUBROUTINE INPA(ANMAT,HANMAT,BRV)                                 LEAF     130
C *** INPA READS AND PRINTS THE NUCLEAR DATA                            LEAF     131
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION ANMAT(NNP,2),HANMAT(NNP,4),BRV(NBR),HBRP(3,NBR)         LEAF     133
      HJ=(7H       )                                                    LEAF     134
      I=1                                                               LEAF     135
      IBR=1                                                             LEAF     136
C *** READ NUCLEAR DATA                                                 LEAF     137
   10 READ 110, (HANMAT(I,J),J=1,4),(ANMAT(I,K),K=1,2)                  LEAF     138
      IF (HANMAT(I,1).EQ.HJ) GO TO 70                                   LEAF     139
      DO 20 J=3,4                                                       LEAF     140
      IF (ABS(HANMAT(I,J))-I) 20,20,30                                  LEAF     141
   20 CONTINUE                                                          LEAF     142
      GO TO 40                                                          LEAF     143
   30 PRINT 120, I                                                      LEAF     144
      CALL EXIT                                                         LEAF     145
C *** TEST FOR BRANCHING RATIOS                                         LEAF     146
   40 DO 60 J=3,4                                                       LEAF     147
      IF (HANMAT(I,J)-0.0) 50,60,60                                     LEAF     148
   50 READ 130, BRV(IBR)                                                LEAF     149
      HBRP(1,IBR)=ABS(HANMAT(I,J))                                      LEAF     150
      HBRP(2,IBR)=I                                                     LEAF     151
      HBRP(3,IBR)=BRV(IBR)                                              LEAF     152
      IBR=IBR+1                                                         LEAF     153
   60 CONTINUE                                                          LEAF     154
      I=I+1                                                             LEAF     155
      GO TO 10                                                          LEAF     156
   70 I=I-1                                                             LEAF     157
      IBR=IBR-1                                                         LEAF     158
      IF (NSKIP.EQ.1) GO TO 100                                         LEAF     159
C *** PRINT DECAY DATA                                                  LEAF     160
      PRINT 140                                                         LEAF     161
      PRINT 150                                                         LEAF     162
      PRINT 160                                                         LEAF     163
      PRINT 150                                                         LEAF     164
      PRINT 170                                                         LEAF     165
      LCNT=13                                                           LEAF     166
      DO 80 J=1,I                                                       LEAF     167
      PRINT 180, (HANMAT(J,JJ),JJ=1,4),(ANMAT(J,JJ),JJ=1,2)             LEAF     168
      LCNT=LCNT+1                                                       LEAF     169
      IF (LCNT.GE.60) PRINT 140                                         LEAF     170
   80 CONTINUE                                                          LEAF     171
      LCNT=LCNT+IBR+8                                                   LEAF     172
      IF(LCNT.GT.60) PRINT 140                                          LEAF     173
C *** PRINT BRANCHING RATIOS                                            LEAF     174
      IF (IBR.EQ.0) GO TO 100                                           LEAF     175
      PRINT 150                                                         LEAF     176
      PRINT 190                                                         LEAF     177
      DO 90 J=1,IBR                                                     LEAF     178
      PRINT 200, HBRP(1,J),HBRP(2,J),HBRP(3,J)                          LEAF     179
   90 CONTINUE                                                          LEAF     180
  100 CONTINUE                                                          LEAF     181
      RETURN                                                            LEAF     182
```

```
C                                                                       LEAF      183
  110 FORMAT (A7,3F4.2D12.5)                                            LEAF      184
  120 FORMAT (1H0,4X,*THERE IS AN ERROR IN NUCLIDE *,I3)                LEAF      185
  130 FORMAT (6D12.5)                                                   LEAF      186
  140 FORMAT (1H1)                                                      LEAF      187
  150 FORMAT (/////)                                                    LEAF      188
  160 FORMAT (51X,*DECAY CHAINS AND NUCLIDE RELATED DATA*)              LEAF      189
  170 FORMAT (61X,*PARENT*,7X,*DECAY*,/,45X,*NUCLIDE   ID*,4X,*1    2*,5 LEAF      190
     1X,*CONSTANT*,4X,*ATOMIC MASS*,/)                                  LEAF      191
  180 FORMAT (45X,A7,1X,F4,2X,F4,1X,F4,2X,1PD12.5,1X,1PD12.5)           LEAF      192
  190 FORMAT (61X,*BRANCHING RATIOS*,/,58X,*FROM*,5X,*TO*,5X,*RATIO*,/)  LEAF      193
  200 FORMAT (57X,F4,4X,F4,3X,F7.4)                                     LEAF      194
      END                                                               LEAF      195
      SUBROUTINE INPC(INT,CONTIC,TEND,LAMDAV,LAMDAL,SOURCE,HANMAT,ITP,AN LEAF      196
     1MAT)                                                              LEAF      197
C *** INPC READS THE CASE DATA                                          LEAF      198
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2      2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2      3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2      4
      DOUBLE PRECISION LAMDAV(NIT),LAMDAL(NIT)                          LEAF      200
      DIMENSION SOURCE(NNT,NIT), HANMAT(NNP,4), CONTIC(NN), TEND(NIT)   LEAF      201
      DIMENSION HTEND(8), ANMAT(NNP,2)                                  LEAF      202
C *** READ BASIC TIME DATA                                              LEAF      203
      READ 120, INT,ITP,IAC,HTIME                                       LEAF      204
      INT1=INT+1                                                        LEAF      205
      TEND(INT1)=HTIME                                                  LEAF      206
C *** READ INITIAL CONCENTRATION OF CONTAINMENT                         LEAF      207
      READ 130, (CONTIC(J),J=1,I)                                       LEAF      208
C *** READ TIME STEP DATA                                               LEAF      209
      DO 10 J=1,INT                                                     LEAF      210
      READ 130, TEND(J),LAMDAV(J),LAMDAL(J)                             LEAF      211
      READ 130, (SOURCE(M,J),M=1,I)                                     LEAF      212
   10 CONTINUE                                                          LEAF      213
C *** CONVERT CONTIC AND SOURCE TO ATOMS AND ATOMS/SEC                  LEAF      214
      IF (IAC.EQ.0) GO TO 60                                            LEAF      215
      DO 50 M=1,I                                                       LEAF      216
      IF (ANMAT(M,1)) 20,20,30                                          LEAF      217
   20 CF=6.0225D+23/ANMAT(M,2)                                          LEAF      218
      GO TO 40                                                          LEAF      219
   30 CF=3.7D+10/ANMAT(M,1)                                             LEAF      220
   40 CONTINUE                                                          LEAF      221
      CONTIC(M)=CONTIC(M)*CF                                            LEAF      222
      DO 50 J=1,INT                                                     LEAF      223
      SOURCE(M,J)=SOURCE(M,J)*CF                                        LEAF      224
   50 CONTINUE                                                          LEAF      225
   60 CONTINUE                                                          LEAF      226
C *** PRINT INPUT DATA                                                  LEAF      227
      IF (NSKIP.EQ.1) GO TO 110                                         LEAF      228
      PRINT 140                                                         LEAF      229
      PRINT 160                                                         LEAF      230
      PRINT 170                                                         LEAF      231
      PRINT 180                                                         LEAF      232
      LIN1=1                                                            LEAF      233
      LIN2=MIN0(7,INT)                                                  LEAF      234
      LIN2P=LIN2+1                                                      LEAF      235
      LIN=INT/7.0+0.99                                                  LEAF      236
      LNCT=9                                                            LEAF      237
      NLN=6+I                                                           LEAF      238
      DO 100 JK=1,LIN                                                   LEAF      239
      PRINT 190                                                         LEAF      240
      L=LIN2P-LIN1+1                                                    LEAF      241
      L=MIN0(8,L)                                                       LEAF      242
      DO 70 J=1,L                                                       LEAF      243
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      K=LIN1-1+J
      HTEND(J)=TEND(K)
   70 CONTINUE
      LM=L-1
      PRINT 200, (HTEND(J),J=1,LM)
      PRINT 210, (HTEND(J),J=2,L)
      PRINT 220, (LAMDAV(J),J=LIN1,LIN2)
      PRINT 230, (LAMDAL(J),J=LIN1,LIN2)
      PRINT 240
      DO 80 JL=1,I
      PRINT 250, HANMAT(JL,1),(SOURCE(JL,J),J=LIN1,LIN2)
   80 CONTINUE
      LIN1=LIN1+7
      LIN2=LIN2+7
      LIN2=MIN0(LIN2,INT)
      LIN2P=LIN2+1
      PRINT 150
      LNCT=LNCT+NLN+3
      LNTST=60-LNCT
      IF (LNTST-NLN) 90,100,100
   90 PRINT 140
      LNCT=0
  100 CONTINUE
  110 CONTINUE
      RETURN
C
  120 FORMAT (3I4,E12.5)
  130 FORMAT (6D12.5)
  140 FORMAT (1H1)
  150 FORMAT (///)
  160 FORMAT (/////)
  170 FORMAT (62X,*TIME STEP DATA*,/)
  180 FORMAT (59X,*TIME PERIOD IN HOURS*,/)
  190 FORMAT (23X,7(12X,* -*))
  200 FORMAT (1H+,21X,7(8X,F6.1))
  210 FORMAT (1H+,27X,7(8X,F6.1))
  220 FORMAT (/,5X,*FILTRATION REMOVAL RATE*,7(2X,1PD12.5))
  230 FORMAT (/,5X,*LEAKAGE REMOVAL RATE    *,7(2X,1PD12.5))
  240 FORMAT (/,5X,*SOURCE TERMS*)
  250 FORMAT (7X,A7,14X,7(2X,1PD12.5))
      END
      SUBROUTINE MAKEA(ANMAT,HANMAT,BRV,A,BL,LAMDV,LAMDL)
C *** MAKEA CONSTRUCTS THE MAIN SOLUTION MATRIX
      IMPLICIT DOUBLE(A-G,P-Z)
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)
      COMMON /BASIS1/ I,IBR,NSKIP
      DOUBLE LAMDV,LAMDL
      DIMENSION ANMAT(NNP,2),BRV(NBR),A(NN,NN),BB(NNT,NNT),BL(NN,NN)
      DIMENSION HANMAT(NNP,4)
      I2=I*2
      IP=I+1
      DO 10 IK=1,I2
      DO 10 JK=1,I2
      A(IK,JK)=0.0D0
      BL(IK,JK)=0.0D0
   10 CONTINUE
      DO 20 IK=1,I
      DO 20 JK=1,I
      BB(IK,JK)=0.0D0
   20 CONTINUE
      IBR=1
      DO 70 IK=1,I
      DO 60 JK=1,I
```

```
      DO 50 IDX=3,4                                                     LEAF       304
C *** IDENTIFY SOURCE TERMS                                             LEAF       305
      IF (ABS(HANMAT(IK,IDX)).NE.JK) GO TO 50                           LEAF       306
      IF (HANMAT(IK,IDX)) 30,30,40                                      LEAF       307
   30 BB(IK,JK)=BRV(IBR)*ANMAT(JK,1)                                    LEAF       308
      IBR=IBR+1                                                         LEAF       309
      GO TO 50                                                          LEAF       310
   40 BB(IK,JK)=ANMAT(JK,1)                                             LEAF       311
   50 CONTINUE                                                          LEAF       312
      A(IK,JK)=BB(IK,JK)                                                LEAF       313
   60 CONTINUE                                                          LEAF       314
      BB(IK,IK)=ANMAT(IK,1)                                             LEAF       315
      A(IK,IK)=-BB(IK,IK)-LAMDL                                         LEAF       316
      IF (HANMAT(IK,2).GT.0.0) A(IK,IK)=A(IK,IK)-LAMDV                  LEAF       317
   70 CONTINUE                                                          LEAF       318
      DO 80 IK=IP,I2                                                    LEAF       319
      JK=IK-1                                                           LEAF       320
      IF (HANMAT(JK,2).GT.0.0) A(IK,JK)=LAMDV                           LEAF       321
   80 CONTINUE                                                          LEAF       322
      DO 130 IK=1,I                                                     LEAF       323
      JJ=IK-1                                                           LEAF       324
      IL=I+IK                                                           LEAF       325
      IF (JJ.EQ.0) GO TO 120                                            LEAF       326
      DO 110 JK=1,JJ                                                    LEAF       327
      JL=I+JK                                                           LEAF       328
      IF (HANMAT(IK,2)) 90,90,100                                       LEAF       329
   90 A(IK,JL)=BB(IK,JK)                                                LEAF       330
      GO TO 110                                                         LEAF       331
  100 A(IL,JL)=BB(IK,JK)                                                LEAF       332
  110 CONTINUE                                                          LEAF       333
  120 CONTINUE                                                          LEAF       334
      A(IL,IL)=-BB(IK,IK)                                               LEAF       335
  130 CONTINUE                                                          LEAF       336
      DO 140 J=1,I                                                      LEAF       337
      BL(J,J)=LAMDL                                                     LEAF       338
  140 CONTINUE                                                          LEAF       339
      RETURN                                                            LEAF       340
      END                                                               LEAF       341
      SUBROUTINE SOLVER(A,B,D,E,TINCD)                                  LEAF       342
C *** SOLVER EVALUATES D(A), I+A*D(A), AND Z(A)                         LEAF       343
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2       2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2       3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2       4
      DIMENSION A(NN,NN), B(NN,NN), C(NN,NN), D(NN,NN), E(NN,NN)        LEAF       345
      DIMENSION F(NN,NN), G(NN,NN)                                      LEAF       346
      M=20                                                              LEAF       347
      SUM=0.0D0                                                         LEAF       348
      I2=I*2                                                            LEAF       349
      DO 20 J=1,I2                                                      LEAF       350
      DO 10 JJ=1,I2                                                     LEAF       351
      SUM=SUM+A(J,JJ)*A(J,JJ)                                           LEAF       352
   10 CONTINUE                                                          LEAF       353
   20 CONTINUE                                                          LEAF       354
      P=(DLOG(SUM)+2.0D0*DLOG(TINCD))/(2.0D0*DLOG(2.0D0))               LEAF       355
      IF (P) 30,30,40                                                   LEAF       356
   30 NP=1                                                              LEAF       357
      GO TO 50                                                          LEAF       358
   40 NP=P+1.0D0                                                        LEAF       359
   50 CONTINUE                                                          LEAF       360
      T=TINCD/(2.0D0**NP)                                               LEAF       361
      CALL SCALAR(A,T,C)                                                LEAF       362
      DO 70 J=1,I2                                                      LEAF       363
      DO 60 JJ=1,I2                                                     LEAF       364
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      B(J,JJ)=0.0D0                                                     LEAF     365
   60 CONTINUE                                                          LEAF     366
      B(J,J)=1.0D0                                                      LEAF     367
   70 CONTINUE                                                          LEAF     368
C *** CALCULATE D(H) AND Z(H)                                           LEAF     369
      DO 90 J=1,M                                                       LEAF     370
      FM=1.0D0/(M+2.0D0-J)                                              LEAF     371
      CALL SCALAR(B,FM,D)                                               LEAF     372
      CALL MULTI(C,D,E)                                                 LEAF     373
      DO 80 JJ=1,I2                                                     LEAF     374
      E(JJ,JJ)=E(JJ,JJ)+1.0D0                                           LEAF     375
   80 CONTINUE                                                          LEAF     376
      CALL EQUAL(E,B)                                                   LEAF     377
   90 CONTINUE                                                          LEAF     378
      S=1.0D0                                                           LEAF     379
      DO 120 J=1,NP                                                     LEAF     380
      Q=S/2.0D0                                                         LEAF     381
      S=S*2.0D0                                                         LEAF     382
      CALL SCALAR(C,Q,F)                                                LEAF     383
      CALL MULTI(F,B,E)                                                 LEAF     384
      DO 100 JJ=1,I2                                                    LEAF     385
      E(JJ,JJ)=E(JJ,JJ)+1.0D0                                           LEAF     386
  100 CONTINUE                                                          LEAF     387
      CALL MULTI(B,E,F)                                                 LEAF     388
      CALL EQUAL(B,G)                                                   LEAF     389
      CALL EQUAL(F,B)                                                   LEAF     390
C *** D(A)                                                              LEAF     391
      CALL EQUAL(G,E)                                                   LEAF     392
      CALL MULTI(G,E,F)                                                 LEAF     393
      CALL SCALAR(F,0.25D0,G)                                           LEAF     394
      CALL SCALAR(D,0.50D0,E)                                           LEAF     395
      DO 110 JI=1,I2                                                    LEAF     396
      DO 110 JJ=1,I2                                                    LEAF     397
  110 D(JI,JJ)=E(JI,JJ)+G(JI,JJ)                                        LEAF     398
C *** Z(A)                                                              LEAF     399
  120 CONTINUE                                                          LEAF     400
      CALL SCALAR(A,TINCD,F)                                            LEAF     401
      CALL MULTI(F,B,E)                                                 LEAF     402
      DO 130 JJ=1,I2                                                    LEAF     403
  130 E(JJ,JJ)=E(JJ,JJ)+1.0D0                                           LEAF     404
C *** I + A * D(A)                                                      LEAF     405
      RETURN                                                            LEAF     406
      END                                                               LEAF     407
      SUBROUTINE FSOLVE(BL,B,D,E,CONTIC,CSOURC,XNTOT,XIENV,TINCD,XENVIC) LEAF     408
C *** FSOLVE CALCULATES THE FINAL CONCENTRATIONS AND INTEGRALS          LEAF     409
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION B(NN,NN), BL(NN,NN), D(NN,NN), E(NN,NN), F(NN,NN), X(NN) LEAF     411
     1, Y(NN), XENVIC(NN)                                               LEAF     412
      DIMENSION CONTIC(NN),CSOURC(NN),XNTOT(NN),XIENV(NN)               LEAF     413
      I2=I*2                                                            LEAF     414
C *** CALCULATE CONTAINMENT AND FILTER INVENTORY                        LEAF     415
      CALL MVMUL(E,CONTIC,X)                                            LEAF     416
      CALL SCALAR(B,TINCD,F)                                            LEAF     417
      CALL MVMUL(F,CSOURC,Y)                                            LEAF     418
      CALL VADD(X,Y,XNTOT)                                              LEAF     419
C *** CALCULATE INTEGRATED RELEASE                                      LEAF     420
      TINCD2=TINCD*TINCD                                                LEAF     421
      CALL SCALAR(B,TINCD,F)                                            LEAF     422
      CALL MVMUL(F,CONTIC,XIENV)                                        LEAF     423
      CALL SCALAR(D,TINCD2,F)                                           LEAF     424
      CALL MVMUL(F,CSOURC,Y)                                            LEAF     425
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      CALL VADD(XIENV,Y,X)                                              LEAF     426
      CALL MVMUL(BL,X,Y)                                                LEAF     427
      CALL VADD(Y,XENVIC,XIENV)                                         LEAF     428
      DO 10 J=1,12                                                      LEAF     429
      CONTIC(J)=XNTOT(J)                                                LEAF     430
      XENVIC(J)=XIENV(J)                                                LEAF     431
   10 CONTINUE                                                          LEAF     432
      RETURN                                                            LEAF     433
      END                                                               LEAF     434
      SUBROUTINE PREP(XOUT,ANMAT,INT)                                   LEAF     435
C *** PREP CONVERTS DENSITIES TO CURIES OR GRAMS                        LEAF     436
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION XOUT(NNT,6,NIT),ANMAT(NNP,2)                            LEAF     438
      DO 50 J=1,INT                                                     LEAF     439
      DO 40 JJ=1,I                                                      LEAF     440
      IF (ANMAT(JJ,1).EQ.0.0) GO TO 20                                  LEAF     441
      CURIES=ANMAT(JJ,1)/3.7D+10                                        LEAF     442
      DO 10 JX=1,5,2                                                    LEAF     443
      XOUT(JJ,JX+1,J)=XOUT(JJ,JX,J)*CURIES                              LEAF     444
   10 CONTINUE                                                          LEAF     445
      GO TO 40                                                          LEAF     446
   20 GRAMS=ANMAT(JJ,2)/6.0225D+23                                      LEAF     447
      DO 30 JX=1,5,2                                                    LEAF     448
      XOUT(JJ,JX+1,J)=XOUT(JJ,JX,J)*GRAMS                               LEAF     449
   30 CONTINUE                                                          LEAF     450
   40 CONTINUE                                                          LEAF     451
   50 CONTINUE                                                          LEAF     452
      RETURN                                                            LEAF     453
      END                                                               LEAF     454
      SUBROUTINE PAPER(XOUT,ANMAT,HANMAT,INT,ITP,TEND)                  LEAF     455
C *** PAPER PRINTS THE RESULTS OF LEAF                                  LEAF     456
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION XOUT(NNT,6,NIT),HANMAT(NNP,4),TEND(NIT),ANMAT(NNP,2)    LEAF     458
      DO 30 J=1,INT,ITP                                                 LEAF     459
      HTIME=TEND(J)                                                     LEAF     460
      PRINT 40                                                          LEAF     461
      PRINT 50, HTIME                                                   LEAF     462
      PRINT 60                                                          LEAF     463
      PRINT 70                                                          LEAF     464
      PRINT 80                                                          LEAF     465
      DO 20 JK=1,I                                                      LEAF     466
      IF (ANMAT(JK,1).EQ.0.0) GO TO 10                                  LEAF     467
      PRINT 90, HANMAT(JK,1),(XOUT(JK,JT,J),JT=1,6)                     LEAF     468
      GO TO 20                                                          LEAF     469
   10 PRINT 100, HANMAT(JK,1),(XOUT(JK,JT,J),JT=1,6)                    LEAF     470
   20 CONTINUE                                                          LEAF     471
   30 CONTINUE                                                          LEAF     472
      RETURN                                                            LEAF     473
C                                                                       LEAF     474
   40 FORMAT (1H1)                                                      LEAF     475
   50 FORMAT (47X,*FISSION PRODUCT INVENTORY AT *,F7.2,* HOURS*,/)      LEAF     476
   60 FORMAT (29X,80HSTABLE NUCLIDE INVENTORIES ARE GIVEN IN GRAMS AND A LEAF     477
     1RE NOTED BY A * IN THE MARGIN,//)                                 LEAF     478
   70 FORMAT (20X,*NUCLIDE*,9X,*CONTAINMENT INVENTORY*,10X,*FILTER INVEN LEAF     479
     1TORY*,11X,*INTEGRATED RELEASE*,/)                                 LEAF     480
   80 FORMAT (30X,3(8X,*ATOMS       CURIES,GM*),/)                      LEAF     481
   90 FORMAT (20X,A7,4X,3(3X,1PD12.5,1X,1PD12.5))                       LEAF     482
  100 FORMAT (18X,1H*,1X,A7,4X,3(3X,1PD12.5,1X,1PD12.5))                LEAF     483
      END                                                               LEAF     484
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      SUBROUTINE TERM(XOUT,HANMAT,INT,ITP,TEND)                         LEAF     485
C *** TERM CREATES TI 700 OUTPUT FORMAT                                  LEAF     486
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION XOUT(NNT,6,NIT),HANMAT(NNP,4),TEND(NIT)                 LEAF     488
      DO 20 J=1,INT,ITP                                                 LEAF     489
      HTIME=TEND(J)                                                     LEAF     490
      PRINT 30, HTIME                                                   LEAF     491
      PRINT 40                                                          LEAF     492
      DO 10 JK=1,I                                                      LEAF     493
      PRINT 50, HANMAT(JK,1),(XOUT(JK,JT,J),JT=1,5,2)                   LEAF     494
   10 CONTINUE                                                          LEAF     495
   20 CONTINUE                                                          LEAF     496
      RETURN                                                            LEAF     497
C                                                                       LEAF     498
   30 FORMAT (//,2X,*FISSION PRODUCT INVENTORY AT*,F7.2,* HOURS IN ATOMS LEAF     499
     1*,/)                                                              LEAF     500
   40 FORMAT (2X,*NUCLIDE*,3X,*CONTAINMENT INVENTORY*,3X,*FILTER INVENTO LEAF     501
     1RY*,3X,*INTEGRATED RELEASE*,/)                                    LEAF     502
   50 FORMAT (2X,A7,7X,D12.5,10X,D12.5,8X,D12.5)                        LEAF     503
      END                                                               LEAF     504
      SUBROUTINE PRMAT(A,IT)                                            LEAF     505
C *** PRMAT PRINTS THE A MATRIX                                         LEAF     506
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION A(NN,NN)                                                LEAF     508
      IP=I+1                                                            LEAF     509
      I2=I*2                                                            LEAF     510
      N=1                                                               LEAF     511
      PRINT 50,IT                                                       LEAF     512
      PRINT 60,N                                                        LEAF     513
      DO 10 J=1,I                                                       LEAF     514
   10 PRINT 70,(A(J,JJ),JJ=IP,I2)                                       LEAF     515
      N=N+1                                                             LEAF     516
      PRINT 60,N                                                        LEAF     517
      DO 20 J=1,I                                                       LEAF     518
   20 PRINT 70,(A(J,JJ),JJ=1,I)                                         LEAF     519
      N=N+1                                                             LEAF     520
      PRINT 60,N                                                        LEAF     521
      DO 30 J=IP,I2                                                     LEAF     522
   30 PRINT 70,(A(J,JJ),JJ=1,I)                                         LEAF     523
      N=N+1                                                             LEAF     524
      PRINT 60,N                                                        LEAF     525
      DO 40 J=IP,I2                                                     LEAF     526
   40 PRINT 70,(A(J,JJ),JJ=IP,I2)                                       LEAF     527
      RETURN                                                            LEAF     528
C                                                                       LEAF     529
   50 FORMAT(1H1,5X,*A MATRIX PRINTED BY QUADRANTS FOR TIME INTERVAL*,  LEAF     530
     1 I3,/)                                                            LEAF     531
   60 FORMAT(/,5X,*QUADRANT*,I3,/)                                      LEAF     532
   70 FORMAT(2X,12(1X,1PD10.3))                                         LEAF     533
      END                                                               LEAF     534
      SUBROUTINE SCALAR(A,S,B)                                          LEAF     535
C *** SCALAR MULTIPLIES A SCALAR TIMES A MATRIX IN DOUBLE               LEAF     536
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION A(NN,NN), B(NN,NN)                                      LEAF     538
      I2=I*2                                                            LEAF     539
      DO 20 J=1,I2                                                      LEAF     540
      DO 10 JJ=1,I2                                                     LEAF     541
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      B(J,JJ)=S*A(J,JJ)                                                 LEAF     542
   10 CONTINUE                                                          LEAF     543
   20 CONTINUE                                                          LEAF     544
      RETURN                                                            LEAF     545
      END                                                               LEAF     546
      SUBROUTINE MULTI(A,B,C)                                           LEAF     547
C *** MULTI MULTIPLIES TWO MATRICES IN DOUBLE                            LEAF     548
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION A(NN,NN), B(NN,NN), C(NN,NN)                            LEAF     550
      I2=I*2                                                            LEAF     551
      DO 20 K=1,I2                                                      LEAF     552
      DO 20 KK=1,I2                                                     LEAF     553
      AM=0.0D0                                                          LEAF     554
      DO 10 J=1,I2                                                      LEAF     555
   10 AM=AM+A(K,J)*B(J,KK)                                              LEAF     556
   20 C(K,KK)=AM                                                        LEAF     557
      RETURN                                                            LEAF     558
      END                                                               LEAF     559
      SUBROUTINE EQUAL(A,B)                                             LEAF     560
C *** EQUAL SETS A MATRIX EQUAL TO A MATRIX IN DOUBLE                   LEAF     561
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION A(NN,NN), B(NN,NN)                                      LEAF     563
      I2=I*2                                                            LEAF     564
      DO 20 K=1,I2                                                      LEAF     565
      DO 10 KK=1,I2                                                     LEAF     566
      B(K,KK)=A(K,KK)                                                   LEAF     567
   10 CONTINUE                                                          LEAF     568
   20 CONTINUE                                                          LEAF     569
      RETURN                                                            LEAF     570
      END                                                               LEAF     571
      SUBROUTINE MVMUL(A,B,C)                                           LEAF     572
C *** MVMUL DOES PRODUCT OF MATRIX AND VECTOR                           LEAF     573
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION A(NN,NN), B(NN), C(NN)                                  LEAF     575
      I2=I*2                                                            LEAF     576
      DO 20 KI=1,I2                                                     LEAF     577
      AM=0.0D0                                                          LEAF     578
      DO 10 KJ=1,I2                                                     LEAF     579
   10 AM=AM+A(KI,KJ)*B(KJ)                                              LEAF     580
   20 C(KI)=AM                                                          LEAF     581
      RETURN                                                            LEAF     582
      END                                                               LEAF     583
      SUBROUTINE VADD(A,B,C)                                            LEAF     584
C *** VADD DOES VECTOR ADDITION                                         LEAF     585
      IMPLICIT DOUBLE(A-G,P-Z)                                          PARAM2     2
      PARAMETER (NNT=10),(NN=2*NNT),(NIT=25),(NBR=10),(NNP=NNT+1)       PARAM2     3
      COMMON /BASIS1/ I,IBR,NSKIP                                       PARAM2     4
      DIMENSION A(NN), B(NN), C(NN)                                     LEAF     587
      I2=I*2                                                            LEAF     588
      DO 10 KI=1,I2                                                     LEAF     589
      C(KI)=A(KI)+B(KI)                                                 LEAF     590
   10 CONTINUE                                                          LEAF     591
      RETURN                                                            LEAF     592
      END                                                               LEAF     593
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